Increasing Risk Aversion and Asset Price Puzzles[1]

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Abstract

We examine asset returns, equity premium and portfolio allocation in a three-period OLG model with increasing risk aversion (IRA). IRA preferences generate results that are more consistent with U.S. data for the equity premium, level of savings and portfolio shares, without assuming unreasonable levels of risk aversion. We find that the relative difference between the two risk aversions (how much more risk-averse old agents are relative to the middle aged) matters more than the average risk aversion in the economy (how much more risk averse both cohorts are). Our findings are robust with respect to a number of model generalizations.

JEL Classification: G0, G12, D10, E21.

Key Words: Equity premium puzzle, Overlapping generations model, Increasing Risk Aversion, Portfolio allocation.

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1. Introduction

The "equity premium puzzle," first presented in the seminal work of Mehra and Prescott (1985), underscores the inability of standard, reasonably parametrized representative-consumer exchange models to match the historical equity premium observed both in the U.S. and in international markets. Since then, a large body of literature has focused on reconciling the high equity premium observed in the data with the theoretical findings of reasonably specified asset pricing models. Several generalizations of the key features of the Mehra-Prescott (1985) model have been proposed, ranging from preference modifications, lower tail risks, survival bias, incomplete markets, market imperfections, limited participation, macroeconomic shocks, and behavioral explanations.[1] Though enormous progress has been made in reconciling facts with theory, no single unified theory appears to have solved all aspects of the puzzle.

Unresolved (or partially resolved) questions in asset pricing theory are, in a sense, manifestations of unresolved issues in portfolio choice theory, which have also been documented extensively in the literature. Early models based on the dynamic framework of Merton (1969), Mossin (1968), and Samuelson (1969) predict a constant optimal share of the risky asset in the portfolio over the life-cycle, independent of age and wealth and dependent only on the level of risk aversion and on the moments of asset returns. However, when calibrated to historical values of asset returns, predicted optimal portfolio shares of equity holdings derived from these models appear unreasonably high, ranging from 42% in Netherlands to over 100% for Germany (73% for the U.S.) (Jorion and Goetzmann (2000)). This is the portfolio-allocation version of the equity premium puzzle: calibrated at historical (high) levels of equity premium and moderate (known) levels of risk aversion, these models produce counter-factually high demand for equities

[1] See, for example, Rietz (1988), Weil (1989), Constantinides (1990), Epstein and Zin (1990), Telmer (1993), He and Modest (1995), Heaton and Lucas (1997), Shrikhande (1997), Basak and Cuoco (1998), Campbell and Cochrane (1999), Constantinides, Donaldson and Mehra (2002), McGrattan and Prescott (2003), Bansal and Yaron (2004), Lauterbach and Reisman (2004), Barro (2006), DaSilva and Giannikos (2006), Hong and Stein (2007), and DaSilva, Farka and Giannikos (2011).

forcing theoretically optimal portfolios to be much more heavily invested in stocks than what is observed in data.

These discrepancies are further highlighted by a large body of empirical work which has consistently found that the share of the risky asset in household portfolios is considerably below 100%.[2] For example, Bertaut and Starr-McCluer (2002) find that the average share of stocks in financial portfolios in the U.S. is around 54%. Similarly, Gomes and Michaelides (2005) estimate the equity share at 54.8% based on the Survey of Consumer Finances (SCF). Other studies document a pronounced life-cycle pattern of the risky asset share in the portfolio: Fagereng, Gottlieb and Guiso (2013) find that households hold a remarkably stable share of risky assets (around 39%) up until the age of 50, which is then reduced to around 30% by the time of retirement. Andersson (2001) shows that the fraction of risky asset follows a hump-shaped age profile, while the share of the safe asset has a distinct U-shaped pattern.

Not surprisingly, a vast body of work has addressed the portfolio allocation puzzle. Standard models have been extended to analyze asset allocation decisions in both infinite and finite horizon models and include a number of key features such as uninsurable labor income risk, preference heterogeneity, market participation costs, precautionary and retirement savings, bequest motives, small probability of disastrous events, and housing investment.[3] While these works have vastly improved our understanding of the nature of the puzzles, most studies tend to address one key statistic at a time: matching either shares (or participation) or asset returns. In those cases when a number of key statistics are matched simultaneously the models assume either unrealistic wealth accumulation or extreme parameter values.[4]

[2]See, for example, Ameriks and Zeldes (2001), Bertaut and Starr-McCluer (2002), Guiso and Sodini (2012), Gomes and Michaelides (2005).

[3]See, for example, Telmer (1993), Heaton and Lucas (1997), Bertaut and Haliassos (1997), Cocco, Gomes and Maenhout (2005), Cocco (2005), Polkovnichenko (2004), Viceira (2001), Gomes and Michaelides (2003, 2005, 2008), Yao and Zhang (2005), and Storesletten, Telmer and Yaron (2007).

[4]Cocco, Gomes and Maenhout (2005), Gomes and Michaelides (2005), and Ball (2008), to name a few, calibrate the models to realistic levels of equity premium and match asset allocation (and participation rates) in a life-cycle setting, but they do so by assuming high levels of risk aversion

This paper presents an asset pricing model that aims at matching both the observed equity premium and asset allocation within a unified framework. We investigate the equity premium puzzle and portfolio allocations in the overerlapping generations (OLG) framework of Constantinides, Donaldson, and Mehra (CDM) (2002) with borrowing constraints. The novelty of this paper lies in introducing increasing risk aversion (*IRA*) in this setting: agents become more risk-averse as they age. This type of preference heterogeneity is motivated by a large number of empirical and survey-based studies which have documented a strong positive relationship between age and risk aversion.[5] This setup allows us to investigate the impact of *IRA* on security returns, level of savings, and portfolio allocations and compare these findings against the CDM (2002) baseline economy with constant risk aversion.

Following CDM (2002), we assume that there are three age cohorts (young, middle-aged and old), each facing different sources of uncertainty on wage and equity income; the attractiveness of equity depends on the stage of the life-cycle. The young, for whom equity is a "hedge" against future wage shocks, are constrained from participating in securities markets. As in CDM (2002), the borrowing constraint feature of the model increases equity returns (because the middle-aged require a higher premium to hold equity), reduces the risk-free rate (because the young are unable to borrow), and thus increases the equity premium. However, while the introduction of the borrowing constraint in a *CRRA* set-up (as in CDM) vastly improves the performance of the model, it still fails to fully account for key aspects of the data: the predicted equity risk premium falls short of the historical average, the level of savings is higher than observed, and portfolio shares tend to be more heavily skewed towards the risky asset than they are in practice.

These shortcomings are fully resolved once we introduce age-dependent increasing

(between 8-10).

[5]See, for example, Morin and Suarez (1983), Riley and Chow (1992), Bakshi and Chen (1994), Lee and Hanna (1995), Palsson (1996), and Sung and Hanna (1996).

risk aversion (*IRA*) into the standard CDM framework with borrowing constraints. We find that the effects of *IRA* on a life cycle model are significant and have the following implications (relative to the baseline *CRRA* case): 1) an increase in all security returns with equity returns dominating bond returns, 2) a higher equity premium, 3) a reduced level of saving, and 4) a decline in the portfolio share of the risky asset. These results are consistent with the empirical evidence: the equity premium generated by *IRA* is in line with the historical average (6%-7%), the saving profile is more consistent with the macroeconomic evidence, and the portfolio share of the risky assets is in the 40-50% range. The key is that these results are obtained for fairly moderate levels of risk aversion ($2-6$ range). Our results are generally robust with respect to a number of model extensions (scale changes, growth, pension schemes) and the main message from this work – that an OLG model with *IRA* delivers results that better match empirical data – remains essentially unchanged under these alternative specifications.

It is interesting to note that we find that the relative difference between the two risk aversions (how much more risk averse old agents are relative to the middle aged) matters more than the average level of risk aversion in the economy (how much more risk averse both cohorts are). The intuition is fairly straightforward: with *IRA* preferences, the marginal investor (the middle-aged agent) faces a more risk averse risk-profile over the life-cycle than in an otherwise identical economy with *CRRA* preferences. As such, he demands a higher premium for holding equity, saves less, and invests a smaller share of his financial wealth in the risky asset. The upward-sloping risk-profile of the agents over the life-cycle enables *IRA* model predictions to better match empirical observations. In contrast, with *CRRA* preferences, the models deliver more realistic results only if risk averse values are implausibly high.

This paper contributes to the two strands of literature – equity premium puzzle and asset allocation puzzle – and it extends them in several dimensions. First, unlike the majority of other works, we study both the equity premium puzzle and portfolio allocation decisions in a unified framework. Second, the introduction of *IRA* in a three-period OLG framework improves upon previous studies by simultaneously matching the equity premium and portfolio allocation shares observed in the data without assuming unreasonable parameter values. The presence of *IRA* reinforces the impact of the borrowing constraint of CDM (2002) on equity prices, thus delivering a higher equity premium and a smaller share of risky asset for all levels of risk aversion. Third, by incorporating a more realistic life-cycle risk-aversion profile, the model predictions can be viewed as a first attempt at examining the effects of an ageing (more risk-averse) population on asset returns and portfolio allocations.

The rest of this paper is organized as follows. Section 2 outlines the model and calibration. Section 3 presents our main findings: results from our *IRA* model are compared to the *CRRA* baseline model of CDM (2002). Various extensions of the baseline model are presented and discussed in section 4. Concluding remarks are summarized in section 5.

2. Model and Calibration

2.1 OLG with Increasing Risk Aversion

We consider the borrowing constrained version of the three-period OLG exchange economy of Constantinides, Donaldson, and Mehra (2002), where each generation lives as young, middle-aged, and old. Each consumer-generation is modeled by a representative agent in order to focus on across-generation instead of within-generation heterogeneity.[6] There is only one consumption good (perishable at the end of each period); wages, consumption, dividends and coupons payments are quoted in terms of this single consumption good. Two types of securities are traded: a bond and a share of equity. The

[6] Following CDM (2002), we assume within-generation market completeness (i.e., the existence of a complete set of contingent claims through which agents of the same generation can insure against their income shocks), while assuming two types of across-generation market incompleteness: a) consumers cannot trade claims against their future income with consumers from another generation, and b) consumers cannot trade with consumers from an unborn generation.

bond is a default-free 20-year government bond: it pays a fixed coupon of one unit of consumption good in every period in perpetuity. The aggregate coupon payment is b in every period (its supply is fixed at b units) and represents a portion of the economy's capital income. p_t^b is the ex coupon price of bond in period t. The equity is the claim to the net dividend stream $\{d_t\}$: the sum total of all the private capital income (stocks, corporate bonds, and real estate). Similarly, the ex dividend price of equity in period t is p_t^e. The total supply of equity is fixed at one unit.

The consumer born in period t receives a low deterministic wage income w^0 in period t (when young), stochastic wage income w_{t+1}^1 in period $t+1$ (when middle-aged), and zero wage income in period $t+2$ (when old). The young start out with zero endowment of bonds and equity. Faced with current low deterministic wages and uncertain future wages, the young would like to hedge income risk by borrowing against future wages, consuming part of the loan, and investing the rest in equity. However, in the borrowing-constrained version of the CDM (2002) economy, the young are prevented from participating in financial markets because human capital (and low wages) does not constitute adequate collateral for loans due to adverse selection and moral hazard issues.[7] Thus, in the borrowing-constrained economy there exists a rational expectations equilibrium in which the young (who value equity investments the most) do not participate in the bond and equity markets. In contrast, investing in equities does not have the same appeal for the middle-aged cohort: at this stage of the life-cycle their

[7] The borrowing constraint on the young, much as in the CDM (2002) work, is exogenously imposed rather then endogenously determined within the model. A more realistic approach would be to allow for uninsurable, persistent and heteroskedastic labor income shocks which would deter the young consumers from investing in equity (Storesletten, Telmer and Yaron (2007)) or a small probability of a disastrous labor income outcome (Cocco, Gomes and Maenhout(2005)). We abstract from modeling labor income risk (leaving it for future research) in order to retain the basic features of the baseline CDM model while highlighting the role of across-generation *IRA* preferences (relative to *CRRA)*. Nonetheless, the imposition of the borrowing constraint can be motivated on a number of grounds as shown by the vast body of work in the extant literature: one-time fixed participation costs (Gomes and Michaelides (2005)), habit formation (Polkovnichenko (2007), counter-cyclical volatility of idiosyncratic income risk (Constantinides and Duffie (1996), Storesletten, Telmer, and Yaron (2007)), housing expenditures (Cocco (2005), high borrowing costs (Davis, Kubler and Willen (2006)), high long-run correlation between equity returns and labor income (Benzoni, Collin-Dufresne and Goldstein (2007) or simply by low trust in the stock market (Guiso, Sapienza and Zingales (2008)).

wage uncertainty is resolved and consumption is highly correlated with equity income. Nonetheless, faced with zero income in the next period, the middle-aged agent optimally decides to save and invest in a diversified portfolio of stocks and bonds in order to smooth consumption over the life-cycle, purchasing $x^b_{t,1}$ bonds and $x^e_{t,1}$ shares of equity. The old consumers sell their bond and stock holdings and consume the proceeds ($x^b_{t,2} = 0$ and $x^e_{t,2} = 0$).

With this set-up, the three-period OLG model of CDM (2002) has three distinct age cohorts: the borrowing-constrained young, the saving middle-aged and the dissaving old. As argued by Constantinides, Donaldson, and Mehra (2002), the borrowing constraint delivers a higher risk-premium than the unconstrained economy because securities are priced solely by the middle-aged investors for whom equity is not as attractive since fluctuations in consumption in this stage of the life-cycle are driven exclusively by fluctuations in equity income. The higher return in equities demanded by the middle-aged and the lower return in bonds due to the inability of the young to borrow combine for a higher risk premium than in standard models.

The novelty of this paper lies in incorporating increasing risk aversion in this setting. The idea is appealingly simple: with the introduction of age-dependent increasing risk aversion, the marginal investor (the middle-aged agent) now faces a more risk-averse risk-profile than in a *CRRA* economy. Specifically, the consumer born in period t has utility:

$$E\left(\sum_{i=0}^{2}\beta^{i}u\left(c_{t,i},\alpha_{i}\right)|I_{t}\right), \tag{1a}$$

where

$$u(c_{t,i},\alpha_i) = \frac{c_{t,i}^{1-\alpha_i}-1}{1-\alpha_i}, \tag{1b}$$

where I_t is the set of all the information available in period t, $\alpha_i > 0$ is the risk aversion parameter.

Our working assumption is that α_2 (risk aversion when old) is higher than α_1 (risk aversion when middle-aged).[8] This assumption is motivated by a large body of empirical work which has consistently documented an upward-sloping pattern of risk aversion over the life cycle. Some studies base their analysis on survey responses designed to elicit individual risk preferences from survey questions. For instance, Sung and Hanna (1996) analyze responses on risk tolerance of the Survey of Consumer Finances and find that risk tolerance decreases with age. Likewise, Dohmen et al. (2011) elicit risk attitudes using a set of survey questions and find that the proportion of individuals who are relatively unwilling to take risks increases strongly with age. Barsky et al. (1997) conclude that individuals between ages 55-70 are more risk-intolerant than other age cohorts based on survey answers to risky scenarios.[9]

Other studies focus on observed portfolio allocation decisions. First, studies in this area have consistently reported strong life-cycle patterns for stock market participation and stock holdings: a hump-shaped participation profile over the life-cycle, a decline in equity shares as investors approach retirement, and a stock market exit after retirement (Andersson (2001), Fagereng et al. (2011), Guiso and Sodini (2013), Guiso, Haliassos and Jappelli (2002)). A number of other works investigate risk attitudes over the life-cycle from observed portfolio allocation decisions. Morin and Suarez (1983) study the effect of age on households' demand for risky assets and conclude that risk aversion displays a distinct life-cycle pattern, increasing uniformly with age. Likewise, Bakshi and Chen (1994) use U.S. asset allocation data post-1945 and document a strong pattern of increasing risk aversion with age. Riley and Chow (1992) derive risk aversion indices from actual asset allocation and find that risk aversion decreases with age until 65 and

[8] More broadly, $\alpha_2 > \alpha_1 > \alpha_0$, where α_0 is the risk-aversion of the young-cohort. However, because the young are excluded from participating in financial markets due to the borrowing constraint, the two relevant risk aversion parameters in our model are α_1 and α_2. The model introduces some form of limited participation since agents participate in the market in two out of the three periods – as savers when middle aged and as dissavers when old.

[9] A few studies have documented either a constant or a decreasing risk aversion with age, at least up until retirement (see for example, Bellante and Saba (1986) and Wang and Hanna (1997)). Nonetheless, there seems to be a general agreement that risk aversion increases beyond age 65 (retirement age). This is corroborated by a drop in stock market participation rates and a decline in risky portfolio shares for agents older than 65.

then increases significantly. A positive relationship between age and risk aversion is also documented in a number of other studies that investigate household asset allocation choices (Palsson (1996) and Lee and Hanna (1995)).[10]

The rest of the model set-up follows closely the Constantinides, Donaldson and Mehra (2002) framework. Let $c_{t,j}$ denote the consumptions in period $t+j$ $(j = 0,\ 1,\ 2)$ of a consumer born in period t. The representative consumer faces the following budget constraints over his life-cycle:

$$c_{t,0} \leq w^0 \tag{2a}$$

when young,

$$c_{t,1} \leq w^1_{t+1} - x^b_{t,1} p^b_{t+1} - x^e_{t,1} p^e_{t+1} \tag{2b}$$

when middle-aged, and

$$c_{t,2} \leq x^b_{t,1}(p^b_{t+2} + 1) + x^e_{t,1}(p^e_{t+2} + d_{t+2}) \tag{2c}$$

when old. We also require that $c_{t,0} \geq 0$, $c_{t,1} \geq 0$, and $c_{t,2} \geq 0$, thus ruling out negative consumption and personal bankruptcy.

We model the joint process of aggregate income and wages of the middle-aged, (y_t, w^1_t), as a time-stationary probability distribution where the aggregate income y_t is given by: $y_t = w^0 + w^1_t + b + d_t$. In the calibration, y_t and w^1_t assume two values each: y_1, y_2 and w^1_1, w^1_2, so that we have a total of four possible realizations for the pair (y_t, w^1_t) represented by four states ($s_t = j$, where $j = 1, ..., 4$). The 4×4 transition probability matrix is denoted by Π.

[10] It is also possible that older agents may appear to be more risk averse not because of an exogenous attitudinal change towards risky outcomes but because they face larger uncertainty over the remainder of their lifetime relative to other age cohorts (such as pension uncertainty or significantly larger health expenditures). In addition, long-horizon mean reversion in stock returns implies that equities may effectively appear to be riskier for the elderly given their relatively shorter investment horizon. These considerations may prompt the older agents to behave as if they are indeed more risk averse than other age cohorts. The age-dependent value of α_i can certainly be motivated on these bases in addition to the abundant empirical evidence cited above. We are grateful to an anonymous referee for pointing this out.

Market clearing in period t requires that the demand for bonds and equity by the young and the middle-aged consumers equal their fixed supply. Since the young are excluded from participation in the borrowing-constrained economy, the supply of bonds and equity must equal the demand of the middle-aged:[11]

$$x^b_{t-1,1} = b \qquad \text{and} \qquad x^e_{t-1,1} = 1. \tag{3}$$

A stationary rational expectations equilibrium in this economy is a set of consumption and investment choices of consumers born in each period and the bond and stock prices in all periods that maximize consumer expected utility ((1a)-(1b)) and satisfy the market clearing conditions (3). Given the consumption constraints $((2a) - (2c))$, the consumer optimization problem with respect to $x^b_{t,1}$ and $x^e_{t,1}$ yields the following first order conditions:

$$u'(c_{t,1})\, p^b_{t+1} = E(\beta u'(c_{t,2})\,(p^b_{t+2} + 1)|I_t) \tag{4a}$$

and

$$u'(c_{t,1})\, p^e_{t+1} = E(\beta u'(c_{t,2})\,(p^e_{t+2} + d_{t+2})|I_t), \tag{4b}$$

The share of wealth saved/invested by the middle-aged investor, and the relative shares of wealth in bonds and equity are easily derived once the pair of price functions is determined. The share of the total wealth saved and invested by the middle-aged investor is given by

$$\Phi^s_{t,1} = \frac{x^b_{t,1}p^b_{t+1} + x^e_{t,1}p^e_{t+1}}{w^1_{t+1}}, \qquad \Phi^b_{t,1} = \frac{x^b_{t,1}p^b_{t+1}}{w^1_{t+1}}, \qquad \text{and} \qquad \Phi^e_{t,1} = \frac{x^e_{t,1}p^e_{t+1}}{w^1_{t+1}}, \tag{5}$$

where $\Phi^s_{t,1}$ denotes the total share of savings/investments as a proportion of the wage income of the middle-aged, while $\Phi^b_{t,1}$ and $\Phi^e_{t,1}$ denote the relative shares of wealth

[11] One limitation of the baseline *IRA* model is that it assumes a fixed supply of assets (bonds and equity) over long periods of time, which is rather unrealistic. There are a number of ways to remedy this issue, which we defer for future work (allowing for inter-generational trading or extending the model to a production economy).One extension that we explore in this paper that relaxes the fixed-supply assumption is the introduction of exogenous growth: as discussed in Section 4b, the general results of the *IRA* baseline model hold reasonably well under this alternative specification.

invested in bonds and equities, respectively. Likewise, the portfolio allocations of bonds and equity as a proportion of the financial portfolio are given by:

$$\omega_{t,1}^{b} = \frac{x_{t,1}^{b} p_{t+1}^{b}}{x_{t,1}^{b} p_{t+1}^{b} + x_{t,1}^{e} p_{t+1}^{e}} \qquad \text{and} \qquad \omega_{t,1}^{e} = \frac{x_{t,1}^{e} p_{t+1}^{e}}{x_{t,1}^{b} p_{t+1}^{b} + x_{t,1}^{e} p_{t+1}^{e}}, \tag{6}$$

where $\omega_{t,1}^{b}$ and $\omega_{t,1}^{e}$ reflect the portfolio shares of bonds and equities, respectively.

Using market clearing condition (3), and dropping the time subscripts, we can write $4a - 4b$ as:

$$u'(c_1)\, p^b(j) = \beta \sum_{k=1}^{4} (u'(c_2)\, \{p^b(k) + 1\}) \Pi_{jk} \tag{7a}$$

and

$$u'(c_1)\, p^e(j) = \beta \sum_{k=1}^{4} (u'(c_2)\, \{p^e(k) + d(k)\}) \Pi_{jk}, \tag{7b}$$

with

$$c_1 = w^1(j) - bp^b(j) - p^e(j) \tag{8a}$$

and

$$c_2 = b(p^b(j) + 1) + p^e(j) + d(j) \tag{8b}$$

for each state j of the economy. With age-dependent risk aversion, the marginal utilities of the middle-aged and old consumers are respectively $u'(c_1) = c_1^{-\alpha_1}$ and $u'(c_2) = c_2^{-\alpha_2}$. Substituting the dynamic budget constraint and the marginal utilities, we have:

$$\frac{p^b(j)}{(w^1(j) - bp^b(j) - p^e(j))^{\alpha_1}} = \beta \sum_{k=1}^{4} \frac{\{p^b(k) + 1\} \Pi_{jk}}{(b(p^b(k) + 1) + p^e(k) + d(k))^{\alpha_2}} \tag{9a}$$

and

$$\frac{p^e(j)}{(w^1(j) - bp^b(j) - p^e(j))^{\alpha_1}} = \beta \sum_{k=1}^{4} \frac{\{p^e(k) + d(k)\} \Pi_{jk}}{(b(p^b(k) + 1) + p^e(k) + d(k))^{\alpha_2}} \tag{9b}$$

These are the two equations to be estimated. Note that the price pairs $p^b(j)$ and $p^e(j)$ are functions of the two risk aversion parameters (α_1 and α_2), which is the unique feature of the *IRA* set-up.

2.2. Calibration

In order to focus exclusively on the impact of increasing risk aversion on security returns, savings, and portfolio choice, our calibration parameters are set as in Constantinides, Donaldson, and Mehra (2002). The set of parameter values used in the calibration of the model is reported in Table 1. Note that since one period in our model spans 20 years (one generation), all parameters are converted to 20-year values so that the annualized return is defined as the geometric average over a 20-year holding period return (i.e., $(1 + 20 - year\ holding\ period\ return)^{\frac{1}{20}} - 1)$.[12]

Following CDM (2002) we specify the transition matrix of the joint Markov process for the wage income of the middle-aged consumers and the aggregate income as:

$$\begin{bmatrix} & (y_1, w_1^1) & (y_1, w_2^1) & (y_2, w_1^1) & (y_2, w_2^1) \\ (y_1, w_1^1) & \phi & \pi & \sigma & H \\ (y_1, w_2^1) & \pi + \Delta & \phi - \Delta & H & \sigma \\ (y_2, w_1^1) & \sigma & H & \phi - \Delta & \pi + \Delta \\ (y_2, w_2^1) & H & \sigma & \pi & \phi \end{bmatrix},$$

where

$$\phi + \pi + \sigma + H = 1. \tag{10}$$

Nine parameters need to be estimated: $y_1/E(y)$, $y_2/E(y)$, $w_1^1/E(y)$, $w_2^1/E(y)$, ϕ, π, σ, H, and Δ. These are selected to satisfy equation (10) and a set of moment conditions which are calibrated as in CDM (2002) based on historical observations and empirical studies. The key calibrated parameters (summarized in Table 1) are: 1) the average share of income going to labor $(E(w^1 + w^0)/E(y))$; 2) the average share of income going to the labor of the young, $w^0/E(y)$ 3) the average share of income going to interest on government debt, $b/E(y)$; 4) the coefficient of variation of the 20–year wage income of the middle-aged, $\sigma(w^1)/E(w^1)$; 5) the coefficient of variation of the 20-year aggregate income, $\sigma(y)/E(y)$; 6) the 20-year auto-correlation of middle-aged wages $(corr(w_t^1, w_{t-1}^1)$,

[12]This section briefly outlines a few key calibration parameters; a more detailed analysis is provided in the original paper of Constantinides, Donaldson, and Mehra (2002).

7) the 20-year auto-correlation of aggregate income $corr(y_t, y_{t-1})$, and 8) the 20-year cross-correlation of aggregate income and middle-aged wages $corr(y_t, w_t^1)$).[13]

Table 2, from Constantinides, Donaldson, and Mehra (2002), shows the historical mean and standard deviations of the annualized, 20-year holding-period return on the S&P 500 series and on the Ibbotson US Government Treasury Long-Term bond yield. As seen, the real mean equity return is between 6%-6.7%, the mean bond real return is around 1%, and the mean equity premium (that we seek to match) is between 5.3%-6.6%.

3. Results

3.1 The Impact of *IRA* on Asset Returns and Equity Premium

The effects of increasing risk aversion on security returns, equity premium, savings, and portfolio shares are presented in Tables 3-8. We present results for various levels of risk aversions of middle-aged and old agents $\{\alpha_1, \alpha_2\}$, thus calibrating a total of 18 model economies. We consider three different levels of constant risk aversion (2, 4 and 6) (which constitute the CDM (2002) baseline case) and compare them with the increasing risk aversion set-up, where the risk aversion of the old agents is increased by small increments relative to the middle-aged (from 0 through 0.25 with an increment of 0.05).[14]

As a preliminary step, we first take a brief look at security returns under constant risk aversion (*CRRA*) as the *average* level of risk aversion in the economy increases, i.e., as we move from the risk-aversion pair $\{\alpha_1, \alpha_2\} = \{2.00, 2.00\}$ to $\{\alpha_1, \alpha_2\} = \{6.00, 6.00\}$

[13] Results shown in the next section are reported for the following levels of (20-year) autocorrelation and cross-correlation of wages and income: $corr(y_t, w_t^1) = 0.1$ and $corr(w_t^1, w_{t-1}^1) = corr(y_t, y_{t-1}) = 0.1$. However, we calibrated our economies for different sets of correlation pairs (high and low correlation) as follows: $corr(y_t, w_t^1) = 0.1$ and $corr(w_t^1, w_{t-1}^1) = corr(y_t, y_{t-1}) = 0.8$, $corr(y_t, w_t^1) = 0.8$ and $corr(w_t^1, w_{t-1}^1) = corr(y_t, y_{t-1}) = 0.1$, and $corr(y_t, w_t^1) = 0.8$ and $corr(w_t^1, w_{t-1}^1) = corr(y_t, y_{t-1}) = 0.8$. Results from these calibrations are similar to the baseline case of 0.1 reported in the paper. They are suppressed for brevity and are available upon request.

[14] More specifically, we compare the CDM baseline case of, say, $\{\alpha_1, \alpha_2\} = \{2.00, 2.00\}$ to the risk aversion pairs $\{\alpha_1, \alpha_2\} = \{2.00, 2.05\}$, $\{\alpha_1, \alpha_2\} = \{2.00, 2.10\}$, $\{\alpha_1, \alpha_2\} = \{2.00, 2.15\}$, etc.

(Table 3, column 1). Consistent with theory, as the overall level of risk aversion increases, equity return rise, bond return decline, and equity premium increases. The equity premium increases from 2.13% when $\{\alpha_1, \alpha_2\} = \{2.00, 2.00\}$ to 4.67% when $\{\alpha_1, \alpha_2\} = \{6.00, 6.00\}$. This is in line with expectations: more risk averse investors generally require a higher premium to hold risky assets. At the same time, a higher *average* risk aversion also implies a higher demand for bonds, which in turn suppresses equilibrium bond returns. The end result is an increase in equity premium and an increase in bond holdings in the financial portfolio.

Next, we focus on the key innovation: the impact of *IRA* on security returns and equity premium. There are several notable observations across all cases reported in Table 3 when the model is extended to account for increasing risk aversion. First, for each level of risk aversion (2, 4 or 6), the introduction of *IRA* preferences leads to an increase in both equity and bond returns, and a higher equity premium. For example, when comparing the baseline CDM (2002) result with a constant risk aversion of 4 (i.e., $\{\alpha_1, \alpha_2\} = \{4.00, 4.00\}$) to *IRA* pairs $\{4.00, 4.25\}$) , equity returns increase from 7.9% to 12.6%, bond returns increase from 3.9% to 6.3% and equity premium rises from 3.9% to 6.3% (Table 3, panel 2). The higher equity premium with *IRA* obtains because old consumers are more risk averse than middle-aged ones ($\alpha_2 > \alpha_1$) which raises both equity and bond returns relative to the *CRRA* scenario, but equity returns increase by more than bond returns given equity's higher risk. The intuition for these results is fairly straightforward: with *IRA*, the (middle-aged) agents become even more averse to gambles that play out in the future (when old) so they save less and consume more compared to the *CRRA* scenario. A lower level of savings means that the overall wealth invested in financial market (both in equities and bonds) also declines. On balance, the effect is to increase both equity and bond returns while increasing the equity risk premium.

The second observation is that IRA preferences tend to have a different effect on security returns than the borrowing constraint. In the case of equity, *IRA* preferences reinforce the effect of the borrowing constraint thus producing higher equity returns. Equity returns are higher in a borrowing constraint economy (relative to an unconstrained economy) because prices are driven exclusively by middle-aged agents for whom equity does not have as much appeal because consumption is highly correlated with equity income. *IRA* preferences further reinforce this effect: as the marginal investor (the middle aged) faces a more risk averse risk-profile over his life-cycle he requires higher equity returns (relative to bonds) in order to hold stocks given its uncertain future payoffs.

In contrast, *IRA* preferences have the opposite impact on bond returns than the borrowing constraint. The imposition of the constraint lowers bond returns in equilibrium because the young cannot borrow at the risk-free rate to invest in equity. With *IRA* preferences, however, the middle-aged agents (who face a more risk-averse life-cycle risk-profile) are now even less willing to defer current consumption. Therefore they save less and demand higher returns on bonds (and equity). This can be illustrated more directly by analyzing the stochastic discount factor (SDF) in the *IRA* economy which can be written as:

$$m_{t+1} = \frac{\beta c_{t,2}^{-\alpha_2}}{c_{t,1}^{-\alpha_1}} = \beta \left(\frac{c_{t,2}}{c_{t,1}} \right)^{-\alpha_1} c_{t,2}^{(\alpha_1 - \alpha_2)}. \tag{11}$$

The first term of (11) $\left(\beta \left(\frac{c_{t,2}}{c_{t,1}} \right)^{-\alpha_1} \right)$ denotes the standard SDF with *CRRA* preferences while the new term $\left(c_{t,2}^{(\alpha_1 - \alpha_2)} \right)$ is added due to the presence of increasing risk aversion. Because in our model middle-aged investors become more risk averse as they age ($\alpha_1 < \alpha_2$), the new term decreases the standard SDF, implying that agents are less willing to shift consumption over time, thus resulting in a higher risk-free rate.[15]

Overall, the model delivers equity premium values that are consistent with their

[15] This is an undesirable feature of the model as it exacerbates the risk-free puzzle. Separating time and risk preferences is one common way to address this issue. DaSilva and Farka (2014) extend the 3-period OLG model presented in this paper to generalized expected utility preferences (GEU) and find that bond returns are lower under this alternative specification. More importantly, they find that the results presented in this paper hold quite well under GEU preferences and it is the increase in risk aversion with age (and not time preferences) that drives the main results.

historical averages even for relatively low levels of risk aversion as long as preferences display increasing risk aversion. Interestingly, the increasing-risk-aversion profile does not have to be steep: the higher equity premium is obtained by fairly small differences in risk aversion values – for all cases presented α_2 is only marginally higher than α_1.

In fact, our results seem to be driven primarily by the *relative difference* between the two risk aversion parameters (how much more risk averse old agents are relative to the middle aged) rather than the *average* risk aversion in the economy (how much more risk averse *both* cohorts are). Higher risk pemium can be easily generated even when the overall risk aversion in the economy is relatively low, as long as the risk profile of the middle-aged is relatively steep. As seen in Table 4 (panel a), the pair $\{\alpha_1, \alpha_2\} = \{2.00, 2.50\}$ (representing a 50% increase in risk aversion over the life-cycle), produces a higher *equity premium* than $\{\alpha_1, \alpha_2\} = \{6.00, 6.25\}$ (which represents a 4.1% increase in risk aversion), even though the *average* level of risk aversion is much higher in the second case.

Of course, allowing for the same *relative increase* in risk aversion (same steepness of the risk profile) for all economies, a higher premium is observed when the *average level* of risk aversion is higher (Table 4, panel b). For each pair in the table, α_2 is 5% higher relative to α_1. As expected, higher asset returns and equity premium are realized for risk aversion pair $\{\alpha_1, \alpha_2\} = \{6.00, 6.30\}$, followed by $\{\alpha_1, \alpha_2\} = \{4.00, 4.20\}$ and lastly by $\{\alpha_1, \alpha_2\} = \{2.00, 2.10\}$.

3.2 The Impact of *IRA* on Savings and Portfolio Allocation

As a first step to analyzing the consumption/saving decisions, we present the consumption patterns for each age group and the savings of the middle-aged in all states for two preference specifications: the baseline CDM (2002) *CRRA* economy (Table 5, panel a) and the *IRA* economy (Table 5, panel b). In both cases, the consumption of the young is the same across all states (since they simply consume their endowment) and the consumption of the middle-aged is relatively smooth. The consumption of the old-age cohort is quite variable, leading the middle-aged to invest some of their wealth in bonds since bonds are a hedge against future consumption variability. With *IRA* preferences, the middle-aged investors are even less willing to give up some of their current consumption in return for higher future consumption. In fact, they now consume more and save less for the future despite higher bond and equity returns. As expected, the variance of the middle-aged consumption is higher with *IRA* preferences relative to the baseline CDM (2002) *CRRA* economy.

Results for consumptions/savings decisions for various combinations of risk aversion pairs in our model economies are presented in Table 6. The first noteworthy observation here is that in economies with *CRRA* preferences (Table 6, column 1), the level of saving is unrealistically high: savings as a share of income (Φ^s) is around 27%-29% while macroeconomic evidence points to a lower range (around 8%-14%). Second, with *CRRA* preferences, as the *average* risk aversion in the economy increases, the level of savings/investment *increases* modestly. Broadly speaking, there are two opposing forces that determine the level of savings/investment: while more risk averse agents optimally prefer to invest less in risky assets, they are also more prudent and want to accumulate more wealth over the life cycle. Under the constant risk aversion scenario, the wealth effect dominates the risk aversion effect thus moderately increasing the overall level of savings in the economy.

These results change dramatically when increasing risk aversion is introduced, delivering results that are more in line with the empirical evidence. For example, the share of savings declines to a more realistic 12.7% with *IRA* preferences (for risk pair $\{\alpha_1, \alpha_2\} = \{4.00, 4.25\}$). In addition, for any given level of risk-aversion, the level of savings decreases as the agents become more risk averse, suggesting that risk aversion dominates the wealth accumulation effect. For example, savings decline from \$12,513 for risk-pairs $\{\alpha_1, \alpha_2\} = \{4.00, 4.00\}$ to a bit less than half that value (\$5,684) for risk pairs $\{\alpha_1, \alpha_2\} = \{4.00, 4.25\}$. In addition, for all calibrated economies with *IRA* preferences, the lowest savings are found for $\{\alpha_1, \alpha_2\} = \{2.00, 2.25\}$ and the highest for $\{\alpha_1, \alpha_2\} = \{6.00, 6.25\}$ consistent with the view that in the first case, agents become "relatively more" risk averse as they age relative to the second case.

Portfolio allocation shares are summarized in Table 7. A few observations are in order: first, with *CRRA* preferences (Table 7, column 1), an increase in the average level of risk aversion in the economy leads to an increase in bond holdings (Φ^b), and a decrease in equity holdings (Φ^e). This is in line with empirical evidence which shows that a higher level of risk aversion increases the demand for bonds (safer asset) and reduces the demand for equity.

Second (and more relevant to this study), the introduction of *IRA* preferences has a significant impact on portfolio shares. The share of wealth invested in bonds and equity declines in all *IRA* economies (relative to the *CRRA* baseline case) since the overall level of savings has now shrunk. However, the drop in equity investment exceeds that of bonds: for example, when comparing risk aversion pairs $\{4.00, 4.00\}$ to $\{4.00, 4.25\}$ the share of wealth invested in equity declines from 17.5% to 5.7% while the bonds' share decreases from 10.5% to around 7.0%.

Third, the introduction of *IRA* preferences produces portfolio shares that are more in line with data compared to *CRRA*, for each pair of risk aversion parameters. Empirical evidence suggests that the share of risky asset in the portfolio is around 50%[16]. With *CRRA*, the portfolio share of equity in our model ranges from 57% to 78% (depending on the average level of risk aversion in the economy) which exceeds empirical estimates. With *IRA* preferences, equity shares appear more in line with the empirical evidence, ranging from 45% to 53%. The lower share of equity in the portfolio reflects a desire to rebalance the portfolio away from the risky assets and towards the safer ones as agents become more risk averse with age. Investing in bonds is appealing to middle-

[16] See, for example, Poterba and Samwick (2001), Bertraut and Starr-McCluer (2002), Ameriks and Zeldes (2004).

aged investors (despite lower returns) because bonds provide a hedge against future consumption variability.

As a final analysis, we also compute portfolio shares for model economies with different overall levels of risk aversion but same steepness of risk profile from middle-aged to old (Table 8). Similar to *CRRA* preferences presented in Table 7, an increase in the *average* (overall) risk aversion in the economy causes a rebalancing of the portfolio away from equity and towards safer assets. Interestingly, the level of savings/investments *declines* modestly as we move from lower risk aversion pairs towards higher levels of risk aversion. This indicates that in the presence of *IRA*, the wealth accumulation that results from increased prudence at higher levels of risk aversion is dominated by the effect of increasing risk aversion from middle-aged to old.

4. Model Extensions and Robustness

In this section, we perform several sensitivity analyses to evaluate the robustness of our baseline estimates. These tests include specifying alternative economies that incorporate different calibrated scales, exogenous growth, and pension schemes. We find that, overall, our baseline results are robust to these alternative specifications and the central message of this paper – that *IRA* preferences improve the outcomes of a standard OLG model – remains essentially unchanged with these model extensions.

4.1 Scale Effects

A potential issue arising from introducing IRA preferences in a standard OLG model is that the stochastic discount factor depends on the absolute level of consumption. Recall that with IRA, the stochastic discount factor is given by:

$$m_{t+1} = \frac{\beta c_{t,2}^{-\alpha_2}}{c_{t,1}^{-\alpha_1}} = \beta \left(\frac{c_{t,2}}{c_{t,1}} \right)^{-\alpha_1} c_{t,2}^{(\alpha_1 - \alpha_2)},$$

where $c_{t,2}^{(\alpha_1-\alpha_2)}$ is the new term due the introduction of *IRA* preference specification. As such, the SDF depends both on the relative difference between α_1 and α_2,as well as the scale of the economy. This implies that asset returns and equity premium, in particular, may no longer be insensitive to scale with *IRA* preferences as they are under *CRRA*.

To check for the sensitivity of our results with respect to scale, we re-calibrate our economies based on alternative levels of aggregate income ($E(y)$): the average level of income is doubled and then tripled. The results, presented in Table 9, are shown for various risk aversion pairs (representing same relative increase in risk aversion) and are contrasted against the baseline case (Table 9, column 1).[17] The results remain essentially unchanged under this modification. For example, for the pair $\{\alpha_1, \alpha_2\} = \{4.00, 4.20\}$,equity returns increase by 0.30% relative to the baseline case when the scale of the economy doubles and by 0.48% when it triples; bond returns increases by 0.16% and 0.26%, respectively, while the premium increases only by 0.14% and 0.22% under the two alternative scenarios. In fact, doubling or tripling the scale of the economy results only in a marginal change in security returns and equity premium – amounting to less than 5% of the baseline findings for all risk aversion pairs.

4.2 Growth Effects

In our baseline framework, we follow Constantinides, Donaldson and Mehra (2002) and abstract from growth, thereby considering an economy that is stationary in levels. This is in contrast to Mehra and Prescott (1985) which model an economy that is stationary in growth rates and has a unit root in levels. Constantinides, Donaldson and Mehra (2002) point out that the choice of a stationary-in-levels economy was partially motivated by the fact that the model ends up being computationally simpler than a growth-stationary economy, and partially because a no-growth economy is consistent with the zero population growth feature of their model. The authors also argue that the general result of their framework – that a borrowing-constrained OLG model produces

[17] In order to focus more explicitly on the effect of scale, we present results for risk aversion pairs that represent the same relative increase in risk aversion over the life-cycle for all model economies (α_2 is 5% higher than α_1). Results for all our calibrated model economies are in line with the findings presented here and are available upon request.

results more in line with empirical evidence – tends to hold up reasonably well when the model is extended to include growth, because return differential across securities are not much affected by this generalization which means that the equity premium does not change much.

Our decision to abstract from growth in our baseline set-up is motivated by the intent to stay as consistent as possible to the original CDM (2002) framework: our primary focus is in evaluating the importance of consumer preference heterogeneity (as captured by increasing risk aversion) in asset returns, equity premium and portfolio allocation decisions. Nonetheless, modern economies exhibit secular growth which tends to increase the mean returns of financial assets relative to the no-growth alternative, even though real rates of return tend to be stationary. To evaluate the robustness of our results, we specify an analogous model which incorporates growth into the baseline OLG framework with increasing risk aversion (and borrowing constraints)[18].

Broadly speaking, the impact of growth on financial assets can be separated in two types: the *windfall effect* and the *substitution effect*. The windfall effect arises because growth creates a preordained increase in future consumption relative to the present. Investors, who are aware of this future windfall, now require a greater return from all securities in order to postpone current consumption and save for the future. The substitution effect arises from the fact that, with growth, the share of output going into wages of the young increases, and the value of dividends decreases. This means that equity becomes relatively less attractive when compared to the bond, which continues to pay a fixed coupon of 1 unit of consumption good in every period in perpetuity. Thus the substitution effect tends to increase equity returns and decrease bond returns. The overall effect of growth on financial assets, therefore, differs across securities: for equities, the substitution effect magnifies the windfall effect resulting in higher equity

[18] We assume that output and population grow exogenously at a deterministic rate. A more complete model would require specifying a stochastic growth process which can be taken up by future research and is outside the scope of the simple extension analyzed here.

returns compared to the no-growth set-up. For the bond, the substitution effect works in the opposite direction to the windfall effect which means that the overall direction of the change is unknown.

The results from this model generalization are presented in Table 10. We compare the baseline case ($n = 0$) with a growth rate of $n = 2\%$ for a number of risk pairs representing the same relative increase in risk aversion. As expected, with secular growth, equity returns increase as both the windfall effect and substitution effects work in the same direction. Bond returns decline for all our risk pairs, suggesting that the substitution effect overpowers the windfall effect. The increase in equity returns and the decline in bond returns combine for a rise in equity premium. In comparison to the no-growth scenario, growth effects tend to be more pronounced in economies with a higher average level of risk aversion relative to low risk aversion. For example, equity premium is only 0.3% higher in the growth scenario relative to no-growth for risk pairs $\{\alpha_1, \alpha_2\} = \{2.00, 2.10\}$, while that difference widens to 2.1% for the $\{\alpha_1, \alpha_2\} = \{6.00, 6.30\}$ risk pairs.

Additionally, while the introduction of growth does not seem to influence much the overall level of savings in the economy it does have a notable impact on portfolio allocations, shifting wealth away from equities towards bonds. For the $\{\alpha_1, \alpha_2\} = \{4.00, 4.20\}$ risk pairs, the share of wealth invested in equities decreases from 7.4% in the no-growth scenario to 5.6%, while the share of bonds increases from 7.8% to 9.5%. Likewise, equity commands a smaller share of portfolio relative to bonds: the share of risky asset in the portfolio drops from 49% (no-growth) to 37% (with growth), while the share of bonds increases from 51% to 63%. This reflects the fact that, with growth, equity is less attractive to investors because of expected higher consumption (windfall effect) and lower dividends (substitution effect).

4.3 Pension Benefits

In the spirit of Constantinides, Donaldson and Mehra (2002), our baseline model includes the simplifying assumption that old consumers receive zero wage once they retire. This assumption can be relaxed to allow for pension income and social security benefits, which we take up in this section. The introduction of pension benefits may have implications for our baseline findings because pension income affects savings, security returns and equity premium (see among others, Abel (2003), Bohn (1999), Cambell and Nosbusch (2007), Olovsson (2004)). Furthermore, this analysis can also be partly motivated by the fact that the historical value of the equity premium in the U.S. appears to be substantially higher since the introduction of the current U.S. Social Security system (pay-as-you-go – PAYGO) in 1935: Mehra and Prescott (2003) document the equity premium for the U.S. to be 3.92% from 1889-1933, and 8.93% from 1934-2000.

While social security income reduces the need for precautionary savings (because of guaranteed retirement income), its impact on the risk premium is theoretically ambiguous. Bond returns increase because, with pension income, there is less precautionary savings in the economy. However, the behavior of equity returns is less clear-cut. In the presence of pension income, investors effectively hold an implicit second asset (the claim to future social security benefits) which is a relatively safe asset and as such exhibits bond-like features (the *implicit asset* effect). This reduces the need to hold bonds directly in the portfolio while increasing the demand for equities: the end-result is lower equilibrium equity returns and a reduced risk premium. In other words, the presence of social security income effectively makes investors less averse to equity risk, thereby reducing the risk premium they require in equilibrium. There is, however, a second offsetting effect on equity returns: with deterministic pension income, investors must be paid a higher rate of return (in both equities and bonds) to entice them to save (the *income* effect). With higher equity and bond returns, the impact on the equity premium is ambiguous[19].

Our model extension considers a pension scheme similar to the U.S. Social Security system (PAYGO) where benefits to current retirees are financed through taxes on those who work. As in the baseline scenario, consumers born in period t receive (low) deterministic $w^0 > 0$ wage income in period t and stochastic wage income w^1_{t+1} when middle-aged (in period $t+1$). However, in contrast to the baseline set-up when pension income was set to zero, consumers now receive a fixed social security benefit ss^2_{t+2} when old (in period $t+2$). To finance these benefits, the payroll tax rate in period t on the young and the middle-aged (the currently working generations), τ_t, is set so that their payroll tax contributions equal the exogenous benefits, i.e.,

$$\tau_t = \frac{ss^2_{t+2}}{w^0 + w^1_{t+1}}. \tag{12}$$

With this payroll taxes and social security benefits, the budget constraint for the consumer born in period t is now:

$$c_{t,0} \leq w^0(1-\tau_t) \tag{13a}$$

when young,

$$c_{t,1} \leq w^1_{t+1}(1-\tau_t) - x^b_{t,1}p^b_{t+1} - x^e_{t,1}p^e_{t+1} \tag{13b}$$

when middle-aged, and

$$c_{t,2} \leq x^b_{t,1}(p^b_{t+2}+1) + x^e_{t,1}(p^e_{t+2}+d_{t+2}) + ss^2_{t+2} \tag{13c}$$

when old.

Table 10 shows the results of this specification. We consider different payroll tax rates when calibrating our results: the current U.S. payroll tax rate (12.4%), a lower

[19] Campell and Nosbusch (2007) argue that social security income while raising the average return on risky assets also increases return volatility, which in turn forces investors to require a higher premium. Return volatility also increases in our specification with pension income, providing an additional explanation for the higher risk premium we find with this generalization.

rate (6.4%) and a higher rate (15%), in order to assess the effect of social security benefits on equity returns, savings and portfolio allocations.[20] Results are shown for both the *CRRA* case (risk pairs $\{\alpha_1, \alpha_2\} = \{4.00, 4.00\}$) and *IRA* (risk pairs $\{\alpha_1, \alpha_2\} = \{4.00, 4.20\}$). As expected, with social security benefits, the level of precautionary savings drops under both preference specifications, thus increasing the return on the bond. This means that the introduction of pension benefits tends to exacerbate the risk-free puzzle because decreased savings produce a non-negligible rise in the equilibrium bond return. Equity returns also rise in our specification indicating that the primary effect of pension income on security returns comes mostly from the *income* effect (which tends to increase required equilibrium returns on both assets in order to entice investors to save) rather than from the *implicit-asset* effect (which tends to increase the demand for equities, thus lowering its return). Because equity is more risky than bonds, the required equilibrium return on equities increases by more than the bond thus increasing the equity premium. These results tend to be slightly more pronounced in specifications with *IRA*, but the overall impact of the pension income is similar in both types of preference specifications.

Overall, our baseline results are fairly robust to the model generalization with pension income: equity premium increases modestly – from 5.8% with no pension income to 6.6% when the payroll tax rate is set at 12.4%. Likewise, portfolio allocation shares do not vary much: the share of equity declines from 48.9% with no pension income to 46% when payroll taxes are set at 6.4%, 43.9% when taxes are 12.4% and 43.5% when taxes are 15%. The share of bond in the portfolio increases modestly in tandem with the decline in equity allocation.

[20] The payroll tax rates of {0%, 6.4%, 12.4%, 15%) correspond to fixed social security benefit levels of: $ss = \{\$0,\ \$3,936,\ \$7,872,\text{and }\$9,447\}$.

5. Conclusions

This paper addresses long-standing issues in asset pricing literature focusing on the equity premium puzzle and asset allocation puzzle. The novelty of the work lies in introducing preference heterogeneity in the form of (age-dependent) increasing risk aversion in the three-period overlapping generations economy of Constantinides, Donaldson, and Mehra (CDM) (2002) with borrowing constraints. We highlight the effect of *IRA* by assuming that older agents are more risk averse than middle-aged ones while retaining key features of the CDM (2002) framework. This type of preference heterogeneity is motivated by a large number of empirical and survey-based studies which have documented a strong positive relationship between age and risk aversion.

We find that age-dependent *IRA* preferences have important implications for asset prices and portfolio allocations. In particular, the IRA specification produces results that are generally more consistent with U.S. data without assuming unreasonable levels of risk aversion: the equity premium is in line with the historical average (in the 5%-7% range), the level of savings is more consistent with the macroeconomic evidence (around 8%-13%), and portfolio shares better match empirical observations (with the share of the risky asset in the 40%-50% range). The results are robust with respect to a number of model generalization: changes in scale, exogenous growth, and the introduction of a simple pension scheme.

The mechanism through which *IRA* preferences work is fairly straightforward: with increasing risk aversion, the marginal investor (the middle-aged investor in the presence of borrowing constraints) faces a more risk-averse risk-profile over his life-cycle. This upward sloping risk-profile induces him to save less, demand a higher premium for holding equity and allocate a smaller share of wealth in the financial portfolio to the risky assets. More specifically, as agents become more risk averse towards gambles that play out in the future, they consume more and save/invest less. An overall lower investment demand tends to boost equilibrium returns in both equities and bonds, but because equity is more risky than bonds and thus commands a higher increase in return relative to bonds. As a result, the equity premium increases. In addition, the overall share of wealth invested in both equity and bonds declines with the decrease in equity investment exceeding the decline in bond investment. This suggests that IRA reduces not only the overall share of wealth in the financial assets, but tilts the composition of the financial portfolio as wealth is shifted away from the risky asset and into the safer one.

Our findings are driven by fairly small increases in risk aversion values from middle-aged to old. Therefore, what matters the most is the relative difference between the two risk aversion parameters (how much more risk averse old agents are relative to the middle-aged) rather than the *average* risk aversion in the economy (how much more risk averse *both* cohorts are).

This study looks at the effect of increasing risk aversion (*IRA*) within the context of the simple three-period OLG framework of Constantinides, Donaldson and Mehra (2002) with *CRRA* preferences. It should be considered as a first attempt in introducing a type of model generalization that addresses both the equity premium puzzle and portfolio allocation decisions in a unified framework and produces results that are more consistent with the empirical evidence. To facilitate comparisons with the baseline *CRRA* CDM case, we have kept the set-up of the model intentionally similar to theirs. Nonetheless, the model abstracts from some key features that may enrich its results. For example, the lack of labor income risk eliminates the precautionary savings motive which would add more realism to the model. In addition, one limitation of *IRA* preferences is that it exacerbates the risk-free rate puzzle. One interesting generalization would be to introduce uninsurable labor income shocks in this framework or separate the effect of increasing risk aversion from the intertemporal rate of substitution. Alternatively, the relaxation of the borrowing constraint may highlight more fully the role of increasing risk aversion on the level of savings, security returns, and household portfolio behavior.

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Table 1
Parameter Values used in Calibration*

Coefficient of Risk Aversion for Middle-Aged: $\boldsymbol{\alpha_1}$	2, 4, and 6
Coefficient of Risk Aversion for Old Agents: $\boldsymbol{\alpha_2}$	2.05...2.25; 4.05...4.25; 6.05..6.35
Subjective Discount Rate: $\boldsymbol{\beta}$	0.44**
Average Aggregate Income: **E(y)**	$98,399
Average Share of Income going to Labor: $\mathbf{E(w^0+w^1)/E(y)}$	0.65
Average Share of Income going to the Young: $\mathbf{w^0/E(y)}$	0.19
Average Share of Income going to Government Debt: **b/(E(y)**	0.03
Coefficient of Variation of 20-year wage Income: $\boldsymbol{\sigma(w^1)/E(w^1)}$	0.25
Coefficient of Variation of 20-year Aggregate Income: $\boldsymbol{\sigma(y)/E(y)}$	0.20
$Corr(y_t,w^1_t)$; $Corr(y_t,y_{t-1})$; $Corr(w^1_t,w^1_{t-1})$	0.10***
$P_1 = 0.275$; $P_2 = 0.225$; $P_3 = 0.225$; $P_4 = 0.275$	

* Parameters are set on a 20-year basis
** Implies an annual $\beta = 0.96$
*** Results are also available for different sets of correlation structures: $corr(y_t, w_t^1) = 0.1$, and $corr(y_t, y_{t-1}) = corr(w^1_t, w^1_{t-1}) = 0.8$; $corr(y_t, w_t^1) = 0.8$ and $corr(y_t, y_{t-1}) = corr(w^1_t, w^1_{t-1}) = 0.1$; $corr(y_t, w_t^1) = 0.8$ and $corr(y_t, y_{t-1}) = corr(w^1_t, w^1_{t-1}) = 0.8$.

Table 2
Historical U.S. Real Returns

	1/1889-12/1999			*1/1926-12/1999*		
	Equity	**Bond**	**Premium**	**Equity**	**Bond**	**Premium**
Mean	**6.15**	**0.82**	**5.34**	**6.71**	**0.14**	**6.58**
Standard Deviation	13.95	7.40	14.32	15.79	7.25	15.21

This is a replica of Table 1 in Constantinides, Donaldson, and Mehra (2002). It shows the mean and standard deviations of the annualized, twenty-year holding-period return on the S&P 500 total return series and on the Ibbotson U.S. Government Treasury Long-Term bond yield. Real returns are CPI adjusted. The annualized mean return (for both the equity and bond) is defined as the sample mean of the $\log(20-\textit{year holding period return})/20$. The annualized standard deviation of the equity (or bond) return is defined as the sample standard deviation of the $\log(20-\textit{year holding period return})/\sqrt{20}$. The annualized mean equity premium is defined as the difference of the mean return on equity and the mean return on the bond. The standard deviation of the premium is defined as the sample standard deviation of the $[\log(20-\textit{year nominal equity return}) - \log(20-\textit{year nominal bond return})]/\sqrt{20}$.

Table 3
Security Returns and Equity Premiums: CRRA and IRA Preferences

	CRRA	***Increasing Risk Aversion***				
	α_1=2.00 α_2=2.00	α_1=2.00 α_2=2.05	α_1=2.00 α_2=2.10	α_1=2.00 α_2=2.15	α_1=2.00 α_2=2.20	α_1=2.00 α_2=2.25
Mean Equity Return	**6.86%**	**7.89%**	**9.11%**	**10.51%**	**12.07%**	**13.77%**
St. Dev of Equity Return	16.42%	18.29%	20.43%	22.77%	25.23%	27.71%
Mean Bond Return	**4.73%**	**5.30%**	**5.98%**	**6.78%**	**7.70%**	**8.73%**
St. Dev of Bond Return	12.75%	14.22%	15.89%	17.73%	19.66%	21.64%
Mean Equity Premium	**2.13%**	**2.59%**	**3.13%**	**3.73%**	**4.37%**	**5.04%**
St. Dev of Equity Premium	18.24%	20.59%	23.09%	25.60%	27.95%	29.98%

	CRRA	***Increasing Risk Aversion***				
	α_1=4.00 α_2=4.00	α_1=4.00 α_2=4.05	α_1=4.00 α_2=4.10	α_1=4.00 α_2=4.15	α_1=4.00 α_2=4.20	α_1=4.00 α_2=4.25
Mean Equity Return	**7.95%**	**8.69%**	**9.53%**	**10.46%**	**11.50%**	**12.65%**
St. Dev of Equity Return	20.56%	21.96%	23.43%	24.94%	26.49%	28.10%
Mean Bond Return	**3.99%**	**4.33%**	**4.73%**	**5.20%**	**5.73%**	**6.32%**
St. Dev of Bond Return	17.21%	18.35%	19.58%	20.91%	22.31%	23.76%
Mean Equity Premium	**3.97%**	**4.36%**	**4.79%**	**5.26%**	**5.77%**	**6.33%**
St. Dev of Equity Premium	24.01%	25.49%	26.92%	28.23%	29.37%	30.28%

	CRRA	***Increasing Risk Aversion***				
	α_1=6.00 α_2=6.00	α_1=6.00 α_2=6.05	α_1=6.00 α_2=6.10	α_1=6.00 α_2=6.15	α_1=6.00 α_2=6.20	α_1=6.00 α_2=6.25
Mean Equity Return	**8.42%**	**8.96%**	**9.57%**	**10.24%**	**10.99%**	**11.83%**
St. Dev of Equity Return	23.04%	24.12%	25.26%	26.47%	27.75%	29.12%
Mean Bond Return	**3.75%**	**4.02%**	**4.31%**	**4.65%**	**5.02%**	**5.43%**
St. Dev of Bond Return	19.11%	20.13%	21.26%	22.48%	23.81%	25.23%
Mean Equity Premium	**4.67%**	**4.95%**	**5.25%**	**5.59%**	**5.97%**	**6.39%**
St. Dev of Equity Premium	26.70%	27.60%	28.45%	29.24%	29.93%	30.49%

The impact of increasing risk aversion on security returns and on the equity premium, for different levels of middle-aged risk aversion (2, 4, and 6). Results are shown for both CRRA and IRA (increasing risk aversion) preferences.

Table 4
Different Relative Increases in Risk Aversion, Security Returns and Equity Premium

Panel a

	Increasing Risk Aversion	
	α_1=2.00 α_2=2.50	α_1=6.00 α_2=6.25
Mean Equity Return	**23.2%**	**11.83%**
St. Dev of Equity Return	38.6%	29.12%
Mean Bond Return	**15.1%**	**5.43%**
St. Dev of Bond Return	29.9%	25.23%
Mean Equity Premium	**8.2%**	**6.39%**
St. Dev of Equity Premium	34.2%	30.49%

Panel b

		Increasing Risk Aversion	
	α_1=2.00 α_2=2.10	α_1=4.00 α_2=4.20	α_1=6.00 α_2=6.30
Mean Equity Return	**9.11%**	**11.50%**	**12.75%**
St. Dev of Equity Return	20.43%	26.49%	30.59%
Mean Bond Return	**5.98%**	**5.73%**	**5.88%**
St. Dev of Bond Return	15.89%	22.31%	26.71%
Mean Equity Premium	**3.13%**	**5.77%**	**6.87%**
St. Dev of Equity Premium	9.11%	11.50%	30.92%

The impact of the relative difference in risk aversion parameters on security returns and on the equity premium. Panel (a) compares a relatively large increase in risk aversion over the life cycle (α_1=2.00; α_2=2.50) to a relatively small change in risk aversion (α_1=6.00, α_2=6.25) for two different levels of risk aversion (2 and 6). Panel (b) compares results across economies experiencing the *same relative increase in risk aversion*.

Table 5
Consumption and Savings/Investments across Different States

	Panel a *CRRA:* α_1=4.00 α_2=4.00				
	State 1	**State 2**	**State 3**	**State 4**	**Average**
Middle-Aged Consumption	**$38,768**	**$34,430**	**$26,821**	**$27,979**	**$32,137**
Old Consumption	**$60,432**	**$25,168**	**$72,379**	**$31,619**	**$47,262**
Young Consumption	$19,000	$19,000	$19,000	$19,000	$19,000
Savings/Investments	**$17,082**	**$21,420**	**$6,629**	**$5,471**	**$12,513**
Equity Investment	$14,430	$8,683	$2,773	$4,607	$7,813
Bond Investment	$2,652	$12,737	$3,856	$864	$4,700
Mean Equity Return	5.07%	4.93%	11.54%	10.38%	7.95%
Mean Bond Return	3.10%	-1.04%	4.53%	8.53%	3.99%
Mean Equity Premium	**1.97%**	**5.97%**	**7.01%**	**1.85%**	**3.97%**

	Panel b *IRA:* α_1=4.00 α_2=4.25				
	State 1	**State 2**	**State 3**	**State 4**	**Average**
Middle-Aged Consumption	**$49,759**	**$42,505**	**$30,520**	**$32,188**	**$38,966**
Old Consumption	**$49,441**	**$17,093**	**$68,680**	**$27,410**	**$40,433**
Young Consumption	$19,000	$19,000	$19,000	$19,000	$19,000
Savings/Investments	**$6,091**	**$13,345**	**$2,930**	**$1,262**	**$5,684**
Equity Investment	$4,784	$3,484	$782	$985	$2,546
Bond Investment	$1,307	$9,862	$2,148	$277	$3,138
Mean Equity Return	9.69%	7.31%	16.04%	17.20%	12.65%
Mean Bond Return	5.51%	-0.95%	6.23%	13.15%	6.32%
Mean Equity Premium	**4.18%**	**8.26%**	**9.81%**	**4.05%**	**6.33%**

The behavior of consumption, savings, equity/bond investment, and security returns across all four states with CRRA (panel a) and IRA preferences (panel b).

Table 6
Consumption and Saving Decisions: CRRA vs IRA Preferences

	CRRA	***Increasing Risk Aversion***				
	α_1=2.00 α_2=2.00	α_1=2.00 α_2=2.05	α_1=2.00 α_2=2.10	α_1=2.00 α_2=2.15	α_1=2.00 α_2=2.20	α_1=2.00 α_2=2.25
Middle Aged Consumption	$32,288	$35,041	$37,350	$39,205	$40,639	$41,715
Old Consumption	$47,111	$44,358	$42,049	$40,194	$38,760	$37,684
Young Consumption	$19,000	$19,000	$19,000	$19,000	$19,000	$19,000
Savings/Investments	$12,362	$9,609	$7,300	$5,446	$4,011	$2,935
Share of Savings (Φ^S)	**27.7%**	**21.5%**	**16.4%**	**12.2%**	**9.0%**	**6.6%**

	CRRA	***Increasing Risk Aversion***				
	α_1=4.00 α_2=4.00	α_1=4.00 α_2=4.05	α_1=4.00 α_2=4.10	α_1=4.00 α_2=4.15	α_1=4.00 α_2=4.20	α_1=4.00 α_2=4.25
Middle Aged Consumption	$32,137	$33,743	$35,246	$36,629	$37,873	$38,966
Old Consumption	$47,262	$45,656	$44,153	$42,770	$41,526	$40,433
Young Consumption	$19,000	$19,000	$19,000	$19,000	$19,000	$19,000
Savings/Investments	$12,513	$10,907	$9,404	$8,022	$6,777	$5,684
Share of Savings (Φ^S)	**28.0%**	**24.4%**	**21.1%**	**18.0%**	**15.2%**	**12.7%**

	CRRA	***Increasing Risk Aversion***				
	α_1=6.00 α_2=6.00	α_1=6.00 α_2=6.05	α_1=6.00 α_2=6.10	α_1=6.00 α_2=6.15	α_1=6.00 α_2=6.20	α_1=6.00 α_2=6.25
Middle Aged Consumption	$31,590	$32,732	$33,840	$34,908	$35,928	$36,893
Old Consumption	$47,809	$46,667	$45,559	$44,491	$43,471	$42,506
Young Consumption	$19,000	$19,000	$19,000	$19,000	$19,000	$19,000
Savings/Investments	$13,060	$11,918	$10,810	$9,743	$8,722	$7,757
Share of Savings (Φ^S)	**29.2%**	**26.7%**	**24.2%**	**21.8%**	**19.5%**	**17.4%**

This table presents consumption levels of the three-age cohorts under different model economies. It also shows the pattern of savings of the middle-aged. Results are reported for CRRA and IRA preferences.

Table 7
Portfolio Allocations: CRRA vs IRA Preferences

	CRRA	Increasing Risk Aversion				
	α_1=2.00 α_2=2.00	α_1=2.00 α_2=2.05	α_1=2.00 α_2=2.10	α_1=2.00 α_2=2.15	α_1=2.00 α_2=2.20	α_1=2.00 α_2=2.25
Investment in Equity	$9,661	$7,156	$5,104	$3,519	$2,362	$1,557
Investment in Bonds	$2,701	$2,453	$2,196	$1,926	$1,650	$1,378
Equity Share % of Wealth (Φ^E)	21.6%	16.0%	11.4%	7.9%	5.3%	3.5%
Bond Share % of Wealth (Φ^B)	6.0%	5.5%	4.9%	4.3%	3.7%	3.1%
Portfolio Allocation: Equity (ω^E)	**78.2%**	**74.5%**	**69.9%**	**64.6%**	**58.9%**	**53.1%**
Portfolio Allocation: Bond(ω^B)	**21.9%**	**25.5%**	**30.1%**	**35.4%**	**41.1%**	**47.0%**

	CRRA	Increasing Risk Aversion				
	α_1=4.00 α_2=4.00	α_1=4.00 α_2=4.05	α_1=4.00 α_2=4.10	α_1=4.00 α_2=4.15	α_1=4.00 α_2=4.20	α_1=4.00 α_2=4.25
Investment in Equity	$7,813	$6,486	$5,287	$4,226	$3,311	$2,546
Investment in Bonds	$4,700	$4,421	$4,117	$3,796	$3,466	$3,138
Equity Share % of Wealth (Φ^E)	17.5%	14.5%	11.8%	9.5%	7.4%	5.7%
Bond Share % of Wealth (Φ^B)	10.5%	9.9%	9.2%	8.5%	7.8%	7.0%
Portfolio Allocation: Equity (ω^E)	**62.4%**	**59.5%**	**56.2%**	**52.7%**	**48.9%**	**44.8%**
Portfolio Allocation: Bond (ω^B)	**37.6%**	**40.5%**	**43.8%**	**47.3%**	**51.2%**	**55.2%**

	CRRA	Increasing Risk Aversion				
	α_1=6.00 α_2=6.00	α_1=6.00 α_2=6.05	α_1=6.00 α_2=6.10	α_1=6.00 α_2=6.15	α_1=6.00 α_2=6.20	α_1=6.00 α_2=6.25
Investment in Equity	$7,449	$6,594	$5,777	$5,000	$4,269	$3,589
Investment in Bonds	$5,611	$5,324	$5,033	$4,742	$4,453	$4,168
Equity Share % of Wealth (Φ^E)	16.7%	14.8%	12.9%	11.2%	9.6%	8.0%
Bond Share % of Wealth (Φ^B)	12.6%	11.9%	11.3%	10.6%	10.0%	9.3%
Portfolio Allocation: Equity (ω^E)	**57.0%**	**55.3%**	**53.4%**	**51.3%**	**48.9%**	**46.3%**
Portfolio Allocation: Bond (ω^B)	**43.0%**	**44.7%**	**46.6%**	**48.7%**	**51.1%**	**53.7%**

The impact of risk aversion on portfolio shares, portfolio allocation, and the total amount invested in equity and bonds. φ^S is the share of wealth saved/invested; φ^B is the share of wealth invested in bonds; φ^E is the share of wealth invested in equity; ω^B is the portfolio share invested in bonds, ω^E is the portfolio share invested in equity. Results are reported for CRA and IRA preferences.

Table 8
Portfolio Allocations: IRA Preferences with Same Relative Increase in Risk Aversion

	Increasing Risk Aversion		
	α_1=2.00 α_2=2.20	α_1=4.00 α_2=4.40	α_1=6.00 α_2=6.60
Savings/Investments	$4,011	$3,330	$3,226
Investment in Equity	$2,362	$1,096	$830
Investment in Bonds	$1,650	$2,234	$2,396
Share of Savings (Φ^S)	**9.0%**	**7.5%**	**7.2%**
Equity Share % of Wealth (Φ^E)	5.3%	2.5%	1.9%
Bond Share % of Wealth (Φ^B)	3.7%	5.0%	5.4%
Portfolio Allocation: Equity (ω^E)	**58.9%**	**32.9%**	**25.7%**
Portfolio Allocation: Bond(ω^B)	**41.1%**	**67.1%**	**74.3%**

This table presents the effect of risk aversion on portfolio shares, portfolio allocation, and the total amount invested in equity and bonds in model economies with the *same relative increase in risk aversion*. φ^S is the share of wealth saved/invested; φ^B is the share of wealth invested in bonds; φ^E is the share of wealth invested in equity; ω^B is the portfolio share invested in bonds, ω^E is the portfolio share invested in equity. Results are reported for CRA and IRA preferences.

Table 9
Scale Effects: Security Returns and Equity Premium with IRA Preferences

	Increasing Risk Aversion		
	α_1=2.00 α_2=2.10 *E(y)=98,399*	α_1=2.00 α_2=2.10 *E(y)=196,798*	α_1=2.00 α_2=2.10 *E(y)=295,197*
Mean Equity Return	**9.11%**	**9.28%**	**9.39%**
St. Dev of Equity Return	20.43%	20.72%	20.90%
Mean Bond Return	**5.98%**	**6.08%**	**6.15%**
St. Dev of Bond Return	15.89%	16.12%	16.25%
Mean Equity Premium	**3.13%**	**3.20%**	**3.24%**
St. Dev of Equity Premium	23.09%	23.41%	23.59%

	Increasing Risk Aversion		
	α_1=4.00 α_2=4.20 *E(y)=98,399*	α_1=4.00 α_2=4.20 *E(y)=196,798*	α_1=4.00 α_2=4.20 *E(y)=295,197*
Mean Equity Return	**11.50%**	**11.80%**	**11.98%**
St. Dev of Equity Return	26.49%	26.92%	27.17%
Mean Bond Return	**5.73%**	**5.89%**	**5.99%**
St. Dev of Bond Return	22.31%	22.70%	22.93%
Mean Equity Premium	**5.77%**	**5.91%**	**5.99%**
St. Dev of Equity Premium	29.37%	29.63%	29.78%

	Increasing Risk Aversion		
	α_1=6.00 α_2=6.30 *E(y)=98,399*	α_1=6.00 α_2=6.30 *E(y)=196,798*	α_1=6.00 α_2=6.30 *E(y)=295,197*
Mean Equity Return	**12.75%**	**13.16%**	**13.40%**
St. Dev of Equity Return	30.59%	31.22%	31.60%
Mean Bond Return	**5.88%**	**6.08%**	**6.20%**
St. Dev of Bond Return	26.71%	27.33%	27.69%
Mean Equity Premium	**6.87%**	**7.08%**	**7.20%**
St. Dev of Equity Premium	30.92%	31.06%	31.13%

The impact of the change in the scale of the economy on security returns and on the equity premium. Results are shown for increasing risk aversion preferences for model economies that display *the same relative increase in risk aversion*.

Table 10
Growth Effects: Security Returns and Equity Premium with IRA Preferences

	Increasing Risk Aversion					
	α_1=2.00 α_2=2.10 *N=0%*	α_1=2.00 α_2=2.10 *N=2%*	α_1=4.00 α_2=4.20 *N=0%*	α_1=4.00 α_2=4.20 *N=2%*	α_1=6.00 α_2=6.30 *N=0%*	α_1=6.00 α_2=6.30 *N=2%*
Mean Equity Return	**9.11%**	**9.12%**	**11.50%**	**12.34%**	**12.75%**	**14.44%**
St. Dev of Equity Return	20.43%	22.04%	26.49%	33.16%	30.59%	41.73%
Mean Bond Return	**5.98%**	**5.69%**	**5.73%**	**5.29%**	**5.88%**	**5.39%**
St. Dev of Bond Return	15.89%	16.28%	22.31%	22.53%	26.71%	26.96%
Mean Equity Premium	**3.13%**	**3.43%**	**5.77%**	**7.05%**	**6.87%**	**9.05%**
St. Dev of Equity Premium	23.09%	25.47%	29.37%	39.06%	30.92%	47.48%
Savings/Investments	$7,300	$7,264	$6,777	$6,729	$6,858	$6,800
Investment in Equity	$5,104	$4,739	$3,311	$2,495	$2,971	$2,007
Investment in Bonds	$2,196	$2,525	$3,466	$4,234	$3,887	$4,793
Share of Savings (Φ^S)	**16.4%**	**16.3%**	**15.2%**	**15.1%**	**15.4%**	**15.2%**
Equity Share % of Wealth (Φ^E)	**11.4%**	**10.6%**	**7.4%**	**5.6%**	**6.7%**	**4.5%**
Bond Share % of Wealth (Φ^B)	**4.9%**	**5.7%**	**7.8%**	**9.5%**	**8.7%**	**10.7%**
Portfolio Allocation: Equity (ω^E)	**69.9%**	**65.2%**	**48.9%**	**37.1%**	**43.3%**	**29.5%**
Portfolio Allocation: Bond (ω^B)	**30.1%**	**34.8%**	**51.2%**	**62.9%**	**56.7%**	**70.5%**

The impact of growth on security returns, equity premium, portfolio shares, portfolio allocation, and on the total amount invested in equity and bonds. φ^S is the share of wealth saved/invested; φ^B is the share of wealth invested in bonds; φ^E is the share of wealth invested in equity; ω^B is the portfolio share invested in bonds, ω^E is the portfolio share invested in equity. Results are shown for increasing risk aversion preferences for model economies that display *the same relative increase in risk aversion*.

Table 11
Pension Scheme: Security Returns and Equity Premium with IRA Preferences

	CRRA				***Increasing Risk Aversion***			
	α_1=4.00 α_2=4.00 *T=0%*	α_1=4.00 α_2=4.00 *T=6.4%*	α_1=4.00 α_2=4.00 *T=12.4%*	α_1=4.00 α_2=4.00 *T=12.4%*	α_1=4.00 α_2=4.20 *T=0%*	α_1=4.00 α_2=4.20 *T=6.4%*	α_1=4.00 α_2=4.20 *T=12.4%*	α_1=4.00 α_2=4.20 *T=12.4%*
Mean Equity Return	**7.95%**	**9.07%**	**10.44%**	**11.12%**	**11.5%**	**13.5%**	**15.9%**	**17.0%**
St. Dev of Equity Return	20.56%	22.71%	25.35%	26.65%	26.5%	29.8%	33.5%	35.2%
Mean Bond Return	**3.99%**	**4.79%**	**5.86%**	**6.41%**	**5.7%**	**7.2%**	**9.3%**	**10.3%**
St. Dev of Bond Return	17.21%	19.27%	21.78%	22.99%	22.3%	25.4%	28.8%	30.3%
Mean Equity Premium	**3.97%**	**4.28%**	**4.59%**	**4.7%**	**5.8%**	**6.3%**	**6.6%**	**6.7%**
St. Dev of Eq. Premium	24.01%	25.72%	27.52%	28.3%	29.4%	31.1%	32.8%	33.5%
Savings/Investments	$12,513	$9,892	$7,531	$6,619	$6,777	$4,726	$3,085	$2,506
Investment in Equity	$7,813	$6,040	$4,493	$3,912	$3,311	$2,174	$1,355	$1,089
Investment in Bonds	$4,700	$3,852	$3,038	$2,707	$3,466	$2,552	$1,730	$1,417
Share of Savings (Φ^S)	**28.0%**	**22.2%**	**16.9%**	**14.8%**	**15.2%**	**10.6%**	**6.9%**	**5.6%**
Equity Share % of Wealth (Φ^E)	17.5%	13.5%	10.1%	8.8%	7.4%	4.9%	3.0%	2.4%
Bond Share % of Wealth (Φ^B)	10.5%	8.6%	6.8%	6.1%	7.8%	5.7%	3.9%	3.2%
Portfolio Allocation: Equity (ω^E)	**62.4%**	**61.1%**	**59.7%**	**59.1%**	**48.9%**	**46.0%**	**43.9%**	**43.5%**
Portfolio Allocation: Bond (ω^B)	**37.6%**	**38.9%**	**40.3%**	**40.9%**	**51.2%**	**54.0%**	**56.1%**	**56.5%**

The impact of pension income on security returns, equity premium, portfolio shares, portfolio allocation, and on the total amount invested in equity and bonds. φ^S is the share of wealth saved/invested; φ^B is the share of wealth invested in bonds; φ^E is the share of wealth invested in equity; ω^B is the portfolio share invested in bonds, ω^E is the portfolio share invested in equity. Results are reported for CRA and IRA preferences.